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ORIGINAL ARTICLES

Dental arch form related with intraoral forces: PR=C

Allen C. Brader, D.D.S., M.S.
Allentown and Bethlehem, Pa.

Discovering the nature of equilibrium of the natural dentition is of primary importance to orthodontists whose concern is achieving ideal and stable dental arches. That the natural dentition exists in equilibrium with and within the oral environment is supported by observation as well as by careful reasoning. Consider the relative stability observed in dental arch form, even during growth, the anatomic position of the dental arch stabilized between the tongue and the circumoral musculature, and the experiences encountered in moving teeth and stabilizing them following such movement, all strongly supporting the concept that the teeth reside in equilibrium in the undisturbed state.

In 1967, Weinstein[1] described changes in tooth position in response to small localized surface additions where encroachment upon adjacent tissues altered natural pressure phenomena. Upon removal of these additions, the teeth promptly returned in the direction of their origins and, by such behavior, supported the balance-of-forces hypothesis. Intraoral form and function relationships are thus founded on observations of stability which sustain the equilibrium concept which states, in particular, that teeth assume unique positions between the opposing forces of tongue and cheek musculature where a balance of forces is obtained.

Consequently, it appears probable that the particular dental arch form in an individual is a predictable segment of a larger morphologic pattern—a harmonious part of the total craniofacial architecture that is, in detail, a locally determined integrant. If this be the case, one is compelled to search for controlling principles that influence the size and form of the dental arch and for the physical constraints that delimit its construction.

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Although rational mechanisms through which a counterbalance of intraoral forces might occur have been proposed, the equilibrium concept remains essentially unverified and experimentally undocumented; studies reporting electronic recordings of tissue forces disclose that tongue pressures usually exceed lip and cheek pressures, so that it appears safe to conclude that the equilibrium cannot consist of equally opposing environmental pressures. Proffit and Norton[2] recently summarized findings from investigations of pressure phenomena during swallowing as follows:

> . . . this evidence also indicates an insensitivity of dental arch form to pressures during activity . . . . Since no investigator has ever been able to detect anything approaching consistent balance of opposing muscle forces, this simplistic notion of the equilibrium theory of tooth position must be abandoned.

In an effort to reconcile the persistent acceptance of the equilibrium theory despite the increasing weight of evidence contradicting it, a new and different dimension of the problem was sought.

Einstein once expressed the idea that "concepts can acquire content only when they are related, however indirectly, with sensible experience." In this context, the word *sensible* implies susceptibility to detection by the senses; that is to say, capable of physical measurement. This study is essentially an inquiry into the nature of human dental arch form wherein an attempt is made to relate morphologic events with experimentally verifiable physical measurements. The approach is mathematical and makes use of geometric constructions to demonstrate principles by which variable intraoral behavior may acquire conceptual content.

Concern in this initial effort is limited to the typically normal, which may serve as a base line for future study of the abnormal referred to only for the advantage of contrast and support of the probability that variations conform in principle with the general hypotheses. The present study integrates findings of other investigators reported in previous literature, recognizing that consistent agreement between observations in biologic events generates relationships which develop confidence about some kind of interdependence and form the basis for conceptual understanding of physical reality.

To achieve the purposes of this investigation, it was necessary to propose certain working hypotheses along with contingent boundary conditions that may be stated as follows:

1. Dental arch form is made up of teeth which assume unique positions along a compound curve representing an equilibrium at all points and delimited by the counterbalancing force fields of the tongue and of the circumoral tissues.
2. The geometry of the curve of dental arch form is best approximated by a *closed* curve with the curvilinear properties inherent in the trifocal ellipse,* the teeth occuping only a portion of the total curve, at its constricted end.

*The trifocal ellipse, unlike the simple ellipse derived from two internal foci, is generated from three internal foci and comprises a closed, compounded elliptic curve as illustrated in Fig. 6.

3. The tissue forces in the resting state are the primary determinants of arch form morphology in contradistinction to the intermittent forces of muscles in functioning states.

A dental rationale and arch form

Each tooth constitutes a structural unit; each exists as an individual entity subject to the physical constrictions of its oral residence. A tooth is independently variable in shape, size, and mass, as well as in site of origin and positional behavior following eruption. It is probably fair to assume that each tooth reacts individually to the force systems that it encounters. For example, isolated teeth and even single teeth maintain positional integrity somewhere along the curve of the dental arcade. When additional individual teeth erupt in ensuing stages of physiologic development, it is conceivable that variable interactions between the teeth (for example, approximal contacts) can contribute additional influences to a priori force systems. In any event, stability of the denture reflects the balanced states of all of the teeth, considered together. In summation, it is the collective response of all of the teeth to all of the environmental forces and their resultant positions in the oral milieu that are observed and described as dental arch form.

Eventually, teeth depart individually, until at last, when the final tooth is gone, as in the beginning before any teeth erupt, there is present only the curvilinear form determined by the environment. In short, arch form exists in the absence of teeth as surely as it exists in the absence of occlusion in certain interarch malocclusions. This train of reasoning generates the conception of a biologically determined curve, along and within which teeth may be considered to be transient residents.

However, the identification of dental arch form conventionally relies upon the presence of the teeth in situ for evidence of the algebraic summation and balance of all the forces to which they are subjected. In natural arch arrangements, then, the teeth themselves reveal, *by the commonality of their collective positions,* a permissible curve defining limits of an environmentally controlled equilibrium. It is this curve, changing with growth and varying with the forces of environmental tissues that I shall attempt to describe in a form and function context.

Arch form morphogenesis; mathematical description

Arch form is generally described in the dental literature as some kind of conic section curve or, alternately, as a catenary curve. Such descriptions are qualitative and often refer to the dental arch as a parabola, an ellipse, or in part as a segment of a circle, or an arc of a sphere. In recent years, gleaning from the works of Mac Conaill and Scher,[3] Scott,[4] and Burdi and Lillie,[5] it appears that the dental profession has been led to consider the mathematic morphogenesis of the dental arch to derive from the catenary curve. Efforts to quantitate this view, however, remain obscure and without either convincing experimental support or mathematic satisfaction. A disclosure by Lu[6] of an attempt to assess arch form reported a mathematic description comprised of

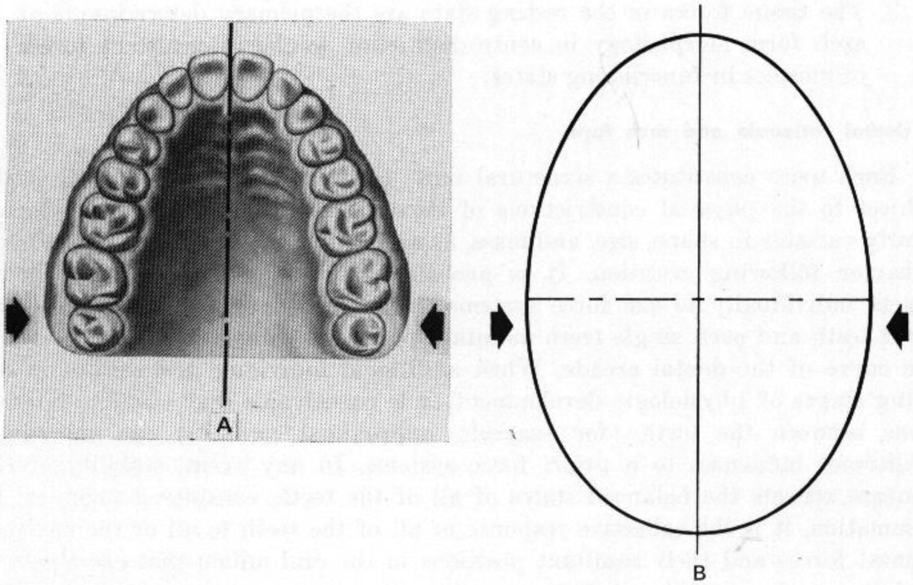

Fig. 1. A, Natural maxillary dental arch exhibiting superior arch form, which illustrates bilateral reflection of the curve of the arch toward the midsagittal plane in the third molar areas. **B,** Form of the compound elliptic curve exhibiting similar bilateral reflections.

polynomials of the fourth exponential power, but without a determinate mean sample pattern.

The development of an elliptic curve followed a series of studies undertaken within the last 5 years by graduate students at Temple University on the mathematic description of dental arch form. A sample of twenty-five superior occlusions taken from the untreated cases selected by Downs[7] were projected by standardized x-ray techniques and were then evaluated by computer analysis. In several studies the sample was compared against the generic equations of known curves for variance of the goodness of fit. I cite the investigations of Currier[8], Garn,[9] and Nielans,[10] each of whose work contributed to the conclusion that the "normal" dental arches of the sample more closely approximated curves with elliptic properties than they did the parabola, the catenary, or other curves which might reasonably describe the curvilinear arch of the teeth. This finding was particularly consistent for the curve describing the outer (facial) surfaces of the maxillary teeth (See Fig. 1, *A* and *B*).

A curvilinear geometry to describe dental arch form

The geometry of the elliptic curves that closely approximate the natural dental arch forms of superior maxillary and mandibular dentures evolved from general mathematical considerations as well as from the findings of the studies cited above which employed computer analysis of the Downs sample. The multiple morphologic variations observed in arch form that, nonetheless, must be believed to conform in principle to the mechanisms of equilibrium exhibit similarities in form that strongly suggest generic associations inherent in a family

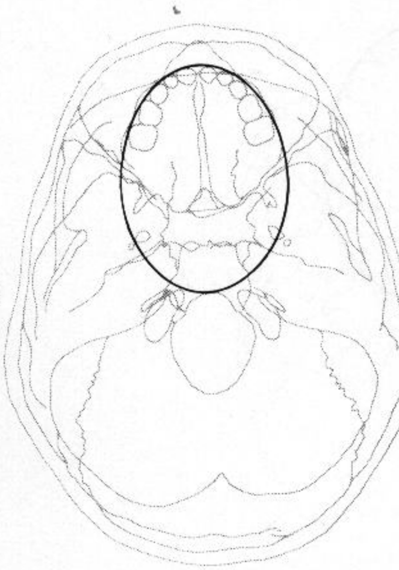

Fig. 2

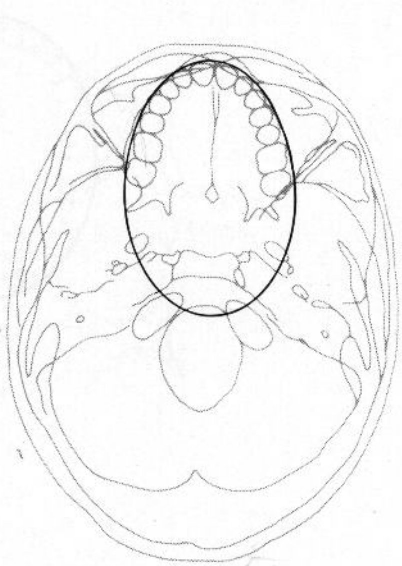

Fig. 3

Fig. 2. Tracing of an x-ray film showing the occlusal view of the maxillary denture of a 5-year-old skull with superior occlusion and arch form. A closed elliptic curve is superposed to disclose the goodness of fit of the curve with the labial and buccal surfaces of the teeth.

Fig. 3. A tracing of an x-ray film of the occlusal view of the maxillary denture of a 15-year-old skull with superior occlusion and arch form. The superposed elliptic curve illustrates agreement with the labial and buccal surfaces of the teeth.

of curves with a common geometric heritage. For example, the generic equation of the simple ellipse is given as:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

in a rectilinear coordinate system. This equation permits the morphogenesis of geometric variations that range from the straight line to the circle as extreme representations.[11] Similarly, the curves of "normal" dental arches may vary within the observed range of morphologic variations as representations of a singular mathematical expression. With this possibility in mind. and with appreciation for recognized principles of biologic continuity, I have developed experimentally a *closed curve* which, in geometric principle, permits a first approximation of the curves of the dental arches and, as such, serves as a geometric model of dental arch form. The proposed curve is an elliptic variation whose mathematic delineation is still being explored. Figs. 2, 3, and 4 exemplify its goodness of fit.

The concept of a closed curve to describe superior arch form has inherent appeal for several important reasons. Regarding the dental arch as an open curve, such as the parabola or the catenary, there is no known method for relating arch widths to arch lengths except as a parameter of tooth sizes and tooth locations along the arch. This method has inherent frailties which become

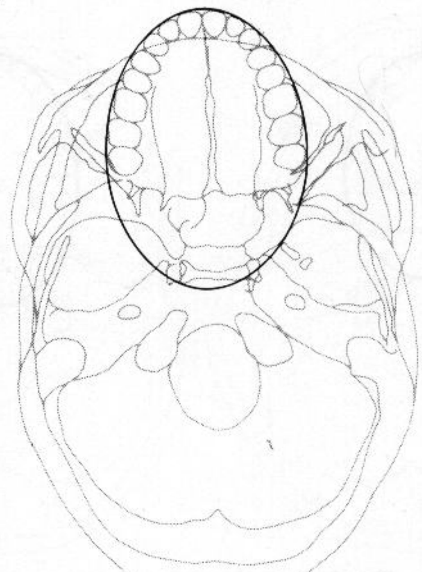

Fig. 4. A tracing of an x-ray film of the occlusal view of the maxillary denture of an adult skull estimated to be 25 years of age or older with superior occlusion and arch form. The superposed elliptic curve demonstrates generalized goodness of fit with the facial surfaces of the teeth.

apparent when one recalls that tooth sizes are independent variables and are known to differ in dimension even between antimeres. Consequently, it can be readily demonstrated that tooth-borne measurement of arch form dimensions (for example, arch lengths and widths) reflect some measure of variation in tooth size and of tooth positions along the curve but are not specific parameters describing the curve itself (Fig. 5). Thus, studies comparing conventional arch form measurements may be subject to unintended experimental errors. For the same reasons, present methods for determining arch lengths for orthodontic diagnosis would appear to measure the teeth largely in relation to themselves.

It is of greater significance, perhaps, that the closed elliptic curve provides a geometric model of arch form with distinct utilitarian advantages. For example, a scaled closed curve has measurable centers and internal foci which are landmarks for comparison of serial or cross-sectional material (Fig. 6).

Quantification of the equilibrium concept

Let us now consider dental concepts of balance and equilibrium, the outcomes of a steady state of energy spent and work performed. There have been three principal methods of investigation of the forces of intraoral equilibrium: (1) tissue distension measurements, (2) electromyographic studies, and (3) electrodynamographic studies employing strain gauge techniques.

Of these, the strain gauge method appears more promising, and the reports of Abrams,[12] Jacobs and Brodie,[13] Jacobs,[14] Winders,[15, 16] Kydd,[17] Proffit and

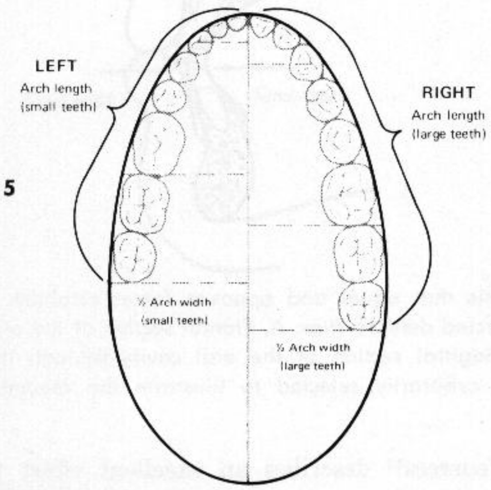

Fig. 5

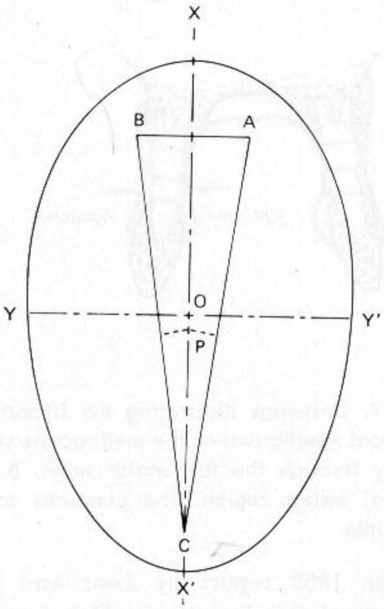

Fig. 6

Fig. 5. A typically average mandibular elliptic arch form curve divided in the midsagittal plane (drawn to scale). The right half of the curve contains large teeth and the left half contains small teeth, illustrating the extremes of the range of observed tooth dimensions. Compare right and left measurements of arch lengths and arch widths, made to identical tooth locations; dimensions must vary as a function of tooth size and do not represent measurements at referable sites along the dental curve. Such measurements are not of the curve itself, but of the teeth.

Fig. 6. A closed compound elliptic curve which serves as a geometric model of arch form. **A, B,** and **C** are the internal foci from which the trifocal curve is generated. **O** is the internal center of the total curve, and **P** is a center between the foci. **X, X′** is the major axis, and **Y, Y′** is the minor axis of the curve. Lines **AC, BC,** and **AB** are the major axes of three simple ellipses interacting to form this compound curve.

associates,[18, 19] Gould and Picton,[20, 21] Lear,[22] and Lear and Moorrees[23] are among those which must be cited.

These investigators examined hypotheses postulating equilibrium mechanisms in various states of function and in the several malocclusion conditions. The most primitive assumption tested by several studies was that illustrated in Fig. 7, where equal and opposite forces were presumed to establish physical balance across selected denture sites. No supportive documentation has been reported for this traditional view, however, because the lingual pressures consistently exceeded the labial pressures under study.

A more subtle approach theorizes that time enters the equation in such fashion as to achieve a balance of all muscular resting and functional forces. Expressed in general mathematic terms, this viewpoint involves pressure (P) and time (t) and may be illustrated as follows:

$$[\Sigma\,(+\,\mathrm{Pa}\ t) + (+\,\mathrm{Pb}\ t) + \ldots] + [\Sigma\,(-\,\mathrm{Px}\ t) + (-\,\mathrm{Py}\ t) + \ldots] = 0$$

where, a, b, c . . . represent positive forces, and x, y, z . . . represent negative forces, diametrically opposite in direction.

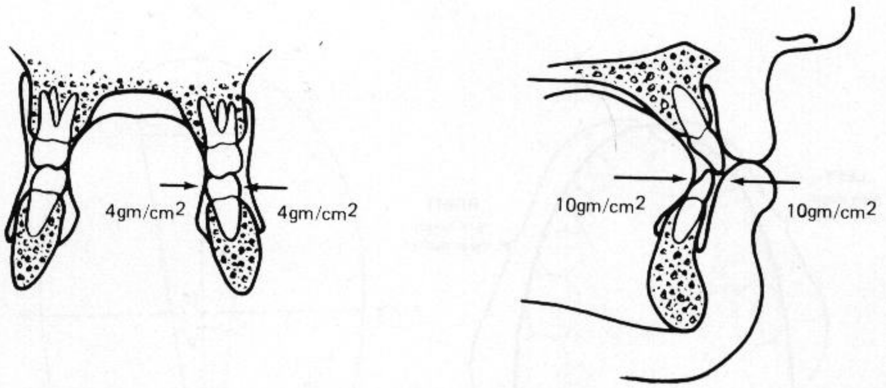

Fig. 7. Drawings illustrating the hypothesis that equal and opposite forces establish a physical equilibrium of the teeth across selected denture sites. **A,** Frontal section of the oral cavity through the first molar teeth. **B,** Sagittal section of the oral cavity through the central incisor region. The pressures are arbitrarily selected to illustrate the assumed principle.

The 1969 report by Lear and Moorrees[23] describes an excellent effort to document this hypothesis. But, in essence, their careful study yielded data that did not support the concept of time-pressure equilibrium between comparable sites across the denture. The lip and cheek pressures they recorded remained lesser than tongue pressures, even though each was multiplied by its respective times of functional activities.

Lear and Moorrees discussed their findings by saying:

> The results of this preliminary study on estimates of 24-hour muscle forces on the dental arches indicate that in a few instances there is justification for the century-old hypothesis that "normal" occlusion is associated with over-all counterbalance between tongue and cheek activity. Yet in the majority of premolar regions lingual force predominated . . . . The enigma of the relationship between dental arch form and muscle function remains.

Clinical experiences and the equilibrium concept

The foregoing evidence seems to be of notable importance to speech pathologists engaged in muscle training related to orthodontics. The rationale for such training, if based upon the assumption that strong, frequent tongue-thrusting of short duration influences the teeth and supporting osseous structures, runs counter to Lear and Moorrees' experimental evidence as well as to general orthodontic experience. In this regard, the muscle-training concept, as assumed, opposes the orthodontist's understanding of the pace of histologic responses in bone induced by mechanical pressures and, in a practical clinical sense, ignores his knowledge of the duration of time required to achieve measurable tooth movement by the application of light continuous forces. Heavy intermittent forces can move teeth, of course, as in the instance of cervical traction appliances where each force application period is relatively long, but the effects of short-time application with poor cooperation are well known, and in such instances the relative times of activity are barely referable with resting forces in total

Fig. 8

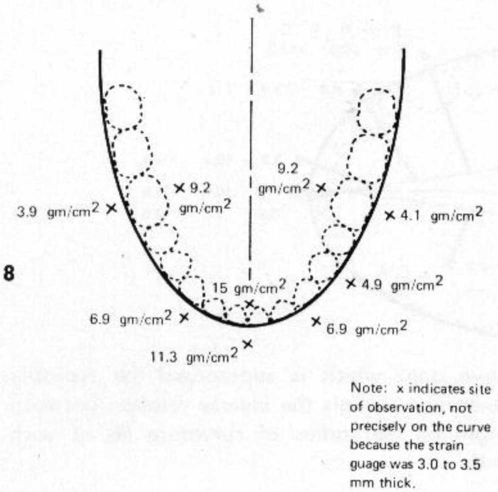

Fig. 9

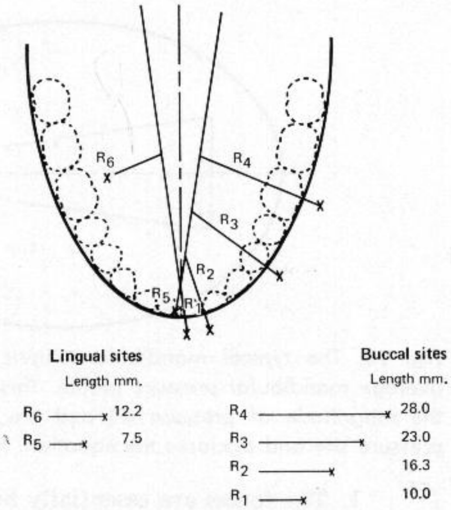

Fig. 8. Typical mandibular pressure profile. The array of recorded tissue pressure measurements at approximate sites of observation. (Winders'[24] data.)

Fig. 9. Typical mandibular dental arch curve (to scale) illustrating successive changes in length of the radius vectors along the curve.

time of activity. In short, strong but briefly intermittent forces, such as those generated by tongue function, may dissipate with little effect for lack of time for quantitative osseous response, *while the light but durable resting forces carry the burden of morphologic determination.*

If this be the case, as reason suggests, it becomes necessary to develop a rationale to amplify the mechanisms through which such forces achieve and maintain an equilibrium with respect to the teeth. There is little doubt about the merit of the basic notion of equilibrium, and present studies are well designed to examine that concept. However, it appears that something remains to be said about the physical principles by which intraoral equilibrium can occur. To propose an extension of present knowledge, I would like to invite attention at this time to (1) de-emphasis of the role played by functional forces, on the one hand, and (2) re-emphasis of the role that the anatomic contours of the environment may contribute to balance on the other.

An intraoral pressure profile method

The data published by previously cited investigators employing electrodynamographic techniques are varied in both kind and degree, and the pressure measurements appear different in scale, which makes correlation difficult. Still a consistent over-all pattern may be discerned. Selecting from the work of Winders[24] and utilizing his data on resting pressures in keeping with the logic developed herein, an array of those data was made in relation to the sites of Winders' observations within the mouth (Fig. 8). It is thus possible to develop what engineers term a pressure profile from which a consistent pattern of three events can be described:

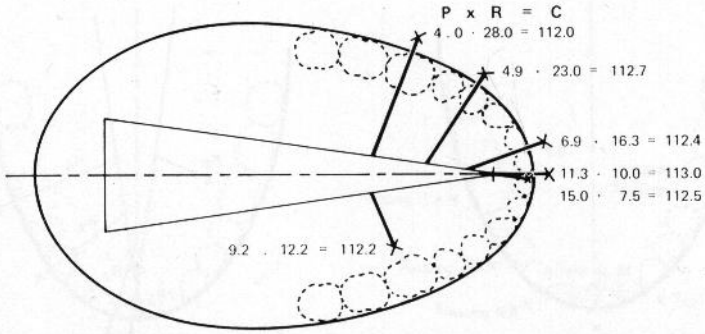

Fig. 10. The typical mandibular elliptic curve onto which is superposed the typically average mandibular pressure profile. This illustration reveals the inverse relation between the magnitude of pressure **(P)** and the length of the radius of curvature **(R)** at each pressure site and discloses the equation: PR=C.

1. The forces are essentially bisymmetric equivalents.
2. The forces are uniformly greater in magnitude internally to the teeth than in the buccal and labial vestibules.
3. The forces diminish progressively, distally from the midsagittal line, on both the buccal and the lingual aspects of the teeth.

Thus, the constructed pressure profile makes apparent an orderly arrangement of pressure magnitudes whose symmetry and pattern logically suggest consistent relations with the form of the typical dental arch with which they exist in juxtaposition.

Having previously achieved an agreement of the closed, elliptic curve with superior natural dental arches, it was compelling to attempt to relate the typical pressure profile with the typical curve by employing a geometric model constructed to *scaled* dimensions (Fig. 9). The purpose was to attempt to correlate with mathematical statement and physical law recorded observations of pressure phenomena with the form of the curve.

Findings from the closed elliptic curve correlated with the intraoral pressure profile

Upon superimposing the tissue pressure profile onto a scaled model of the typically normal mandibular dental arch, it was readily established that a relation existed between the pressure magnitudes and the lengths of the radii of curvature at the pressure sites (Fig. 10). It became obvious that an inverse relation obtained; that is, the greater the pressure, the shorter the radius of curvature. Table I exhibits data to disclose the relations between the pressures and the radii of curvature as illustrated in Fig. 10.

Findings; PR=C

With simple calculation it was determined that along the buccolabial curve of the dental arch, the pressure (P), when multiplied by the radius (R), produces a mathematical constant (C) within close limits of experimental error. The derived equation, PR = C, may be stated and restated by algebraic transformations, as follows:

Table I. Typical measurements for the mandibular denture

Site	*P* (Gm./cm.²)*	*R (mm.)*	*C*
Buccolabial			
At $\overline{6}$	4.0	28	112.0
At $\overline{4}$	4.9	23.0	112.7
At $\overline{3}$	6.9	16.3	112.5
At $\overline{1}$	11.3	10	113.0
Lingual			
At $\overline{6}$	9.2	12.2	112.2
At $\overline{1}$	15.0	7.5	112.5

P, Recorded pressure; R, radius of curvature; C, the mathematical constant produced.

*Winders'[24] data for resting tissue pressures around a typical mandibular denture.

$$(1) \quad PR = C, \text{ or,}$$

$$P = \frac{C}{R}, \text{ and,}$$

$$R = \frac{C}{P}$$

where

P = Pressure in Gm./cm.²

R = Radius of curvature of the elliptic curve at the pressure site in mm.

C = A mathematical constant.

Thus, the equation PR = C expresses the most fundamental associations between forces and shape and reveals an inverse relation between force and curvature; that is to say, the tighter the curve, the greater the pressure per unit area, and the converse would follow.

This finding prompted another—that the pattern of the pressure profile, based on the limited available data, relates generally with known engineering principles that obtain between pressures and the shape of any elastic container. These natural laws are not unknown in biology, where the physical principle of forces across an elastic membrane is described by an equation given by D'Arcy Thompson[25] in his book, *Growth and Form,* in which it is reported that the applicable equation is a product of the work of Laplace in the nineteenth century. The pertinent quotation is given as $Pi = Pe + T\left(\frac{1}{R} + \frac{1}{R}\right)$. This equation describes a unique combination of forces which establish equal and opposing pressures on opposite sides of a curved elastic envelope and maintain it in stable form (shape).

Recognizing the oversimplification, a primitive physical model of intraoral phenomena may be constructed by employing a rubber balloon placed inside a second balloon and having a series of loosely joined solid objects trapped between them. If the inner balloon is then inflated, the solid objects assume positions along curves, the size and form of which are determined by the pressure and tension characteristics of the two balloons. A condition of equilibrium is established along those curves where the outward pressure of the internal balloon is counterbalanced by the inward tension of the external balloon.

Discussion

Perhaps it is important to emphasize that the specific focus of this study is directed to consideration of only two important systems of the total forces believed to influence the positions of the teeth. Nonetheless, the findings establishing possible mathematical agreement between the size and form of the typical dental arch and typical pressure measurements of the environmental tissues deserve further elaboration of the relations described by PR = C.

The tongue as a source of energy internally to the teeth. The hypothesis that PR = C assumes that the tongue is a source of energy, inherent in the (muscle) tissues, and that at any instant in time its resting potential energy is a physical constant (C). Furthermore, this potential energy is expressed against the teeth from within the dental arch, anteriorly and laterally from the tongue source, in a measurable amount (P) which is an inverse function of distance (R), so that

$$P = \frac{C}{R}.$$

While the tonic contractile forces of the tongue muscles may constitute one important source of potential energy, it may be pertinent to the present discussion to direct attention to another possible source of potential energy inherent in the tongue. Anatomically, skeletal muscle tissue is comprised of many muscle cells called muscle fibers, and in the living state the regions between the filaments are filled with a solution of salts and soluble proteins. Therefore, "the tongue can be considered to behave much like a thin-walled rubber tube containing viscous fluids."[27] And, again, "The maximum theoretical work which can be obtained from a muscle is equal to the work actually performed (by contraction) plus the work done by the muscle *against the viscous resistance to change in form.*"[27]

Thus, some expression of the potential energy of the tongue can be credited to resistance to change in tongue form on account of its viscosity and, of course, the tongue's position.

The circumoral tissues as an elastic envelope. Application of the Laplace equation, $\Delta P\left(\frac{RR'}{R+R'}\right) = T$, hypothesizes that the vestibular forces of the cheek and lip tissues against the teeth are inherent in the elastic tensions of the circumoral tissue envelope and that these tensions depend upon the curvilinear shape of the envelope. To develop this conception, it is first necessary to entertain certain *a priori* assumptions:

1. The cheek-lip tissues constitute an elastic envelope; this is a concept that has been attributed to Brodie and is more recently discussed by Graber.[28]
2. The labiobuccal curve of the dental arcade is a smooth, graded curve, suggesting molded determination by yielding, elastic, soft-tissue structures.
3. The pressure *outside* the lips and cheeks is *zero* and is given by: P_e (equal atmospheric pressure on both sides).
4. The resting tissue pressures measured labial and buccal to the teeth

are *internal* pressures, that is, internal to the elastic envelope, and may be labeled P_i.

Then, according to the Laplace equation, defining the equilibrium of pressures across any elastic surface $(Pi - Pe) = \frac{T}{R}$ (eliminating the indeterminate transverse radius, R′, only because its contribution is presently unknown and may be of small magnitude for mathematical reasons)

$$\text{or } (P_i - P_e) = \frac{T}{R}, \text{ and assuming } Pe = 0, \text{ from (3) above, } \therefore P_i = \frac{T}{R},$$
$$\text{or } P_i \cdot R = T.$$

Finding the teeth in equilibrium: C = T. In logical pursuit of the present hypotheses, the equation $P_i \cdot R = T$, which permits calculation of the tension of the elastic envelope, is essentially the same as the equation, PR = C, which permits calculation of the potential energy of the tongue. Because the forces of the tongue (C) are expressed outwardly and away from the tongue, while the forces of the tension of the cheek-lip envelope (T) are expressed inwardly from the cheeks and lips, in diametrically opposite directions, it may be inferred that C = T produces the optimum condition of dental equilibrium at any point along the dental arch form curve.

Mathematically, the balance of the forces of the environmental tissues across the dental arch may be expressed in the following statement of terms:

$$PR = C = T = (Pi - Pe)\left(\frac{RR'}{R + R'}\right)$$

To reiterate, at any selected site along the stable dental arch curve, it may be postulated that the tongue energy constant (C) is equal to the tension constant of the cheek-lip envelope (T), and C = T because the forces are expressed in diametrically opposite directions; although varying in value, C and T remain quantitatively equal at every point along the dental curve (Fig. 11).

The teeth in equilibrium; dual constraints. A long-standing and fundamental tenet of the balance-of-forces concept about dental positions states that the teeth reside in a trough of equilibrium between the tongue forces and forces of the lips and cheeks; that they are thus compelled to occupy (horizontal) positions which are determined by at least two separate sources of (muscular) tissue energy.

From the findings of this study, it is now possible to postulate that the teeth do reside in positions of equilibrium forced anteriorly and laterally by the internal field of forces of the tongue musculature, described by PR = C, and counterbalanced precisely along the interface between the teeth and the labiobuccal mucosa by the inward tension of the enveloping tissues, described by the equation: $(Pi - Pe)\left(\frac{RR'}{R + R'}\right) = T$, or $\Delta PR = T$ (approximately); dental equilibrium obtains wherever C = T.

Clinical support for the idea of dual constraints is abundant, and pathologic entities further substantiate that belief. In general, any factor introducing discontinuity of the soft tissues is capable of disrupting their field of forces. Specific factors altering the force field internal to the teeth (for example, change in tongue volume) would include the extremes of glossotomy on the one

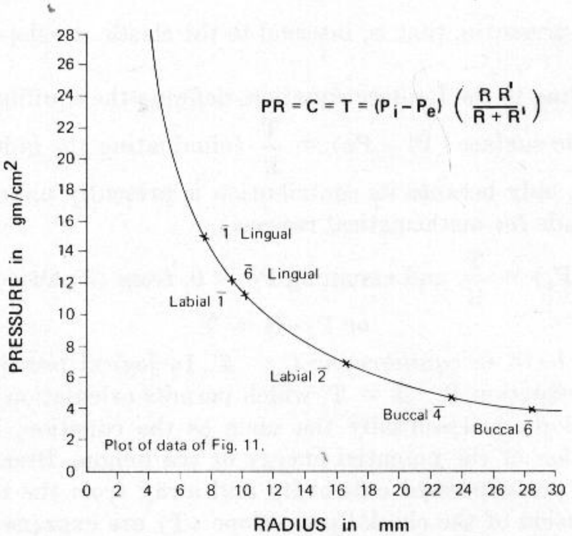

Fig. 11. Graph illustrating the curve of equilibrium of the teeth where **C=T** along the interface between the teeth and the labiobuccal mucosa. Plot is of the data from Table I and demonstrates the inverse relations between the pressures and respective radii.

hand and hyperglossia on the other, with marked changes in tooth positions in anticipated directions. External to the teeth, any alteration of the homogeneity of the elastic envelope of the cheeks and lips, such as occurs in original cleft lip conditions or following surgical closure of cleft lips, could be expected to alter forces and influence dental positions by diminished contraction against the teeth in the former instance and by overcontraction in the latter, with attendant changes in the tension of the envelope and the form of the dental arch.

Physical mechanisms of the equilibrium state, C = T. With an understanding of the relation PR = C, it may be readily appreciated that intraoral pressures progressively diminish concomitantly with increases in the radius of curvature; this inverse relation is illustrated by the graph shown in Fig. 11.

To postulate the mechanics of one situation, let us assume that forces of tooth eruption cause a tooth to enter the force field of the tongue in some position that encroaches upon the tongue. In response, the potential energy of the tongue, at least partly inherent in its resistance to change in form, will influence the tooth directionally away from the tongue, quantitatively a distance R, according to $R = \frac{C}{P}$. The osseous support of the tooth responds to the tongue pressure against the tooth crown and permits positional change of the tooth. Such tooth movement presumes adequate available space, unobstructed by other dental structures or foreign bodies. As the tooth moves away from the tongue, the radius increases and, concurrently, the effective tongue force (P) diminishes in an amount sufficient to approach a state of balance with the force (Pi) of the tension of the elastic envelope of the cheek. Further progress in tooth movement eventually finds the tooth encroaching into the elastic tissue of the buccal or labial envelope. In this event, the envelope experiences a *localized radius* *shortening,* causing Pi, the force of the tension of the envelope, at that site to increase and thus restrict additional movement of the tooth into the envelope. At the exact point where the tongue pressure (P), multiplied by its radius (R), becomes equal to the envelope pressure (P_i), multiplied by its respective radius (R), the condition of equilibrium is satisfied and the position of the tooth becomes stabilized. At this point $C = T$ or, in detail, $PR = C = T = P_i \cdot R$, and the tongue energy balances the tension of the circumoral envelope. Thus, observed dental positions along the arch curve may be considered to reflect the counterbalance between energy conditions of the environmental tissues.

Implications of the finding that PR = C; growth of the dental arches. It has been recognized for many years, as a matter of physiologic principle, that the available energy of a muscle depends upon its mass[25] and the fact that muscle volume increases with normal growth is beyond serious dispute. Writing about the growth of muscle tissues, Bell, Davidson, and Scarborough[27] disclose that "the size of the [muscle fiber] cells and the amount of tissue lying between them, especially elastic fibers, both increase with age." While the tongue is found to be relatively large at birth, subsequent growth nonetheless involves increases in volume from which it follows that the potential energy of the tongue becomes greater with age.

From mathematical considerations, in this article the constant (C), from the equation $PR = C$, has been proposed as a quantitated estimate of the potential energy of the resting tongue muscles. Accepting, then, a quantitative increase in C as a function of growth, it may be understood that $PR = C$ implies generalized increase in values of R that result in *over-all* expansion of the dental arch curve. To reiterate, because of the equality of terms in the equation $PR = C$, any increase in C must be balanced by an increase in R, an increase in P, or an increase in both P and R together.

Weinstein[1] reported that intraoral cheek forces increased with age, and there is abundant skeletal evidence extant to indicate that dental arch dimensions increase with age (Figs. 2, 3, and 4). Thus, there are good reasons to believe that growth of the dental arches involves mechanisms that are accurately described by $PR = C$ and result in both larger arches and greater pressures in older subjects.

The traditional view of growth of the dental arches, with the form considered as an open curve like the catenary, is illustrated in Fig. 12, where superposition of serial events is possible only on the single point they share in common—the median approximal contacts of the central incisor teeth. This viewpoint implies little or no dimensional change in the anterior arc of the arch curve; from such interpretations, restrictive conclusions have been drawn about the movement of teeth during growth[29] and during orthodontic therapy.

Taking advantage of the geometric characteristics of the closed elliptic curve (Fig. 6), and relying on the relations of $PR = C$ as discussed in this section, the mechanisms of growth of the dental arches can be envisioned as *total* curve enlargement about geometric centers. This concept is illustrated in Fig. 13. In comparison with alternate orientations, the closed-curve concept places the dental arches into a different but more satisfying architectural context

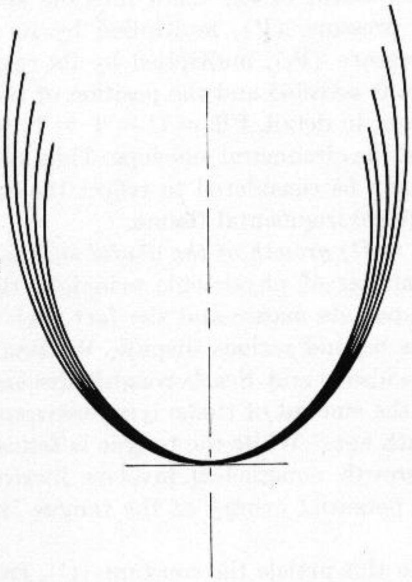

Fig. 12

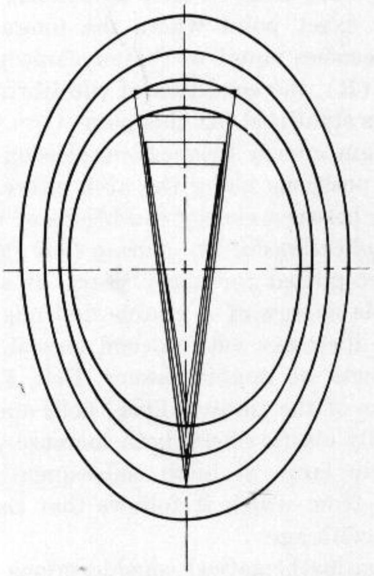

Fig. 13

Fig. 12. Traditional view of dental arch dimension changes with growth of the dentofacial regions. This concept focuses on curves originating at the mesial contact between the central incisor teeth.

Fig. 13. Enlargement of closed arch curves about their geometric centers to illustrate a mechanism for growth changes in the dental arches.

with generally accepted understandings about the growth of the face and the head. But, more important, a total, internally centered curve orientation provides a new method for the reliable comparison of arch forms in both serial and cross-sectional investigations into the nature of growth of the dental arches.

Discussion of clinical implications of the finding that PR = C. In terms of technology, orthodontic treatment consists of rearrangements of widths of teeth within the confines of limiting curves to achieve improved alignment, occlusion, and dentofacial esthetics, along with ideal arch forms and stable dentures. In clinical orthodontic experience, establishing Class I (Angle) occlusal intercuspation (neutroclusion), in and of itself, is not always a sufficient achievement to maintain superior interarch relationships. In an effort to promote understanding about attainable treatment goals, this article discloses new concepts concerning the nature of the equilibrium of the natural dentition and postulates some mechanisms relating arch form with environmental forces.

The findings of this study suggest that dental positions may be expressed within the equation PR = C. The equation PR = C may be restated algebraically as $R = \frac{C}{P}$, which permits mathematic calculation of dental arch form parameters (R) in consonance with resting tissue forces recorded in the individual. Thus, a few measures of intraoral pressures (P) for a patient may be used to calculate the radii of curvature of that curve which represents optimum equilibrium.

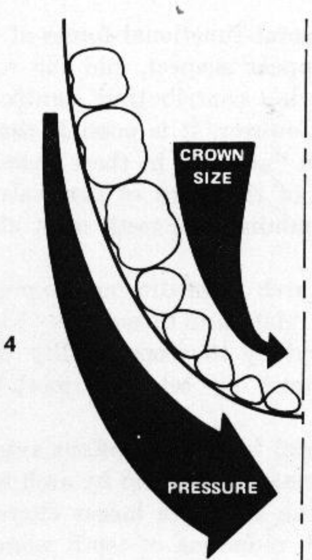

Fig. 14

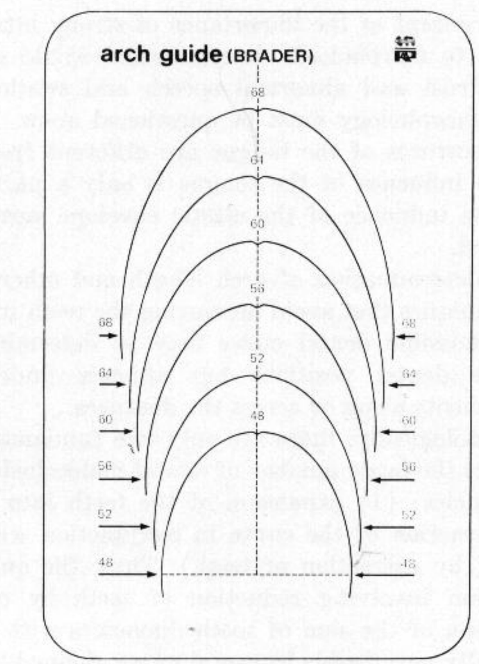

Fig. 15

Fig. 14. In the mandibular denture the smallest tooth crowns erupt at the sites of greatest pressures, and the larger tooth crowns erupt at those locations where pressures are least.
Fig. 15. Typical ideal arch form guides for clinical arch wire constructions. Each curve is mathematically consistent with observation and theory.

Similarily, pressure measurements following treatment (or at any time, for that matter) may be used to calculate the permissible curve and the attainment of balance as the product of orthodontic treatments. Therefore, measurements of intraoral pressures (P) may become necessary diagnostic criteria in the orthodontic evaluation of individual patients.

Since pressure (P) and radius of curvature (R) are inversely related, PR = C "explains" why the mandibular incisor teeth exhibit many crowded positional variations and, of all the teeth in the mouth, the least stability following positional changes produced by orthodontic movements. It is precisely here, in the anterior segment of the mandibular dental arch, where the radius of curvature is smallest (shortest), that the pressures are greatest and therefore exercise the most critical influence on tooth positions (Fig. 10).

It may be worthy of mention that the size of each successive tooth crown along the dental arch appears to vary inversely with the pressure across its location along the dental arch curve. In the mandibular denture in particular, the smaller tooth crowns erupt at the sites of greatest pressures and the larger crowns erupt at those locations where pressures are least (Fig. 14). However, it is only speculation that the environmental tissue forces described by PR = C and ΔPR = T exert some intraosseous influence upon tooth formation that may affect crown dimensions (appreciating the arguments for genetic determination of these structures).

The concept of the importance of strong intraoral functional forces of short duration to morphologic determination would appear suspect, and the role of tongue-thrust and abnormal speech and swallowing contributing significantly to total morphology must be questioned anew. However, it is possible that the resting postures of the tongue are different from "normal" in these cases.* At best, the influence of the tongue is only a part of the story of dento-alveolar form; the influence of the elastic envelope surrounding the teeth must also be considered.

The determination of arch length and other arch form dimensions requires new techniques that avoid measuring the teeth in relation to themselves (Fig. 5). The permissible dental curve may be determined by the commonality of the collective dental positions but appears undefined by selective tooth-borne measurements along or across the dentures.

Technologically, there are only two fundamental treatment options available to correct the large number of dental malocclusions characterized by arch length discrepancies: (1) expansion of the teeth into the arcs of a larger curve and (2) contraction of the curve in conjunction with reduction of tooth mass (for example, by extraction of teeth). Thus, the questions of arch expansion or of contraction involving reduction of teeth by extractions become a matter of comparison of the sum of tooth diameters with the anterior circumference of a biologically permissible intraoral curve, defined by $C = T$. It is not inconceivable that certain of the borderline arch length problems are not susceptible to perfect orthodontic resolution, irrespective of treatment method. Expansion of these cases violates the pressure—radius relation which the muscle constants permit; extraction and contraction of these cases often violate the same physical relations, for mechanical reasons, only in the opposite direction. With this view, only further investigations may resolve the question of which procedure provides the better treatment compromise as judged by the criterion of stable dentitions.

Clinical implications; typical curves as arch form guides. For immediate use in the present state of the art, I have developed typically average curves within the trifocal elliptic family that bracket the range of observed arch sizes and serve as clinically useful arch form guides. These typical curves achieve good over-all fit with the arch forms of the superior occlusion samples tested, which included the dentitions of natural skulls and the casts of the carefully selected Downs[7] series (Downs' cephalometric analysis).

At least four important fabrication requirements appear essential for attaining "ideal" arch wire construction in treatment techniques utilizing labial and buccal appliances: (1) optimum form (shape), (2) correct size, (3) bilateral symmetry, and (4) localized modifications as necessary to accommodate variations in crown morphology. Typical arch guide curves are illustrated in Fig. 15. The techniques of application of the clinical arch guides are readily described as follows:

1. Measure the greatest intermolar width, in millimeters, between the buccal surfaces of the last erupted molar teeth in each arch (excluding the third molars).

*Variations in suspension geometry of the tongue.

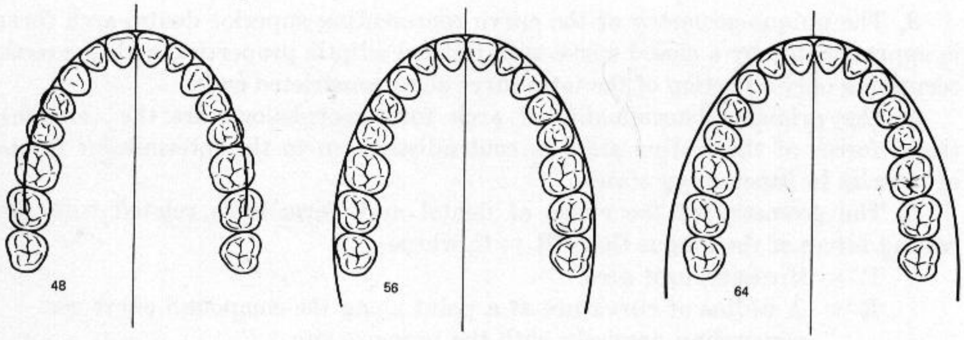

Fig. 16. Illustration of the importance of selecting the properly sized curve for arch wire construction.

2. The width, in millimeters, thus obtained corresponds with the number of the optimum arch curve on the guide chart. Fig. 16 illustrates the importance of selecting the proper-sized curve.

3. The arch wire is shaped to conform closely with the selected arch guide curve by superimposing the wire over the curve, distally from the midline on both sides. The midline point is marked.

4. The arch wire is positioned in the mouth so that the midline mark on the arch wire corresponds with the medial point between the central incisor teeth.

5. The locations are marked and localized modifications are bent for unusual crown morphology variations, if required; these modifications may also include minor bends for the canine eminences, for maxillary lateral incisor setbacks, and for torquing, if desired. Such modifications become more necessary toward the conclusion of treatment, in the finishing arch wires.

6. The (modified) arch wire is superposed to retest the over-all fit with the selected arch guide curve.

7. The arch wire is attached to the teeth with careful attention to preserve the position of midline mark between the central incisor teeth.

Summary and conclusions

A new and different perspective into the nature of the equilibrium of the teeth along the curve of the natural dental arch is made possible by the application of the pressure profile method introduced in this investigation. Although it must be recognized that the proposed hypotheses are incompletely verified and that further investigations are necessary, the findings of the present study appear to support the following conclusions:

1. Dental arch form consists of dental units arranged in unique positions along a compound curve, which represents a steady state of equilibrium delimited by the counterbalancing force fields of the tongue and of the circumoral tissues.

2. The curvilinear geometry of dental arch form can be ascertained by describing the commonality of the collective positions of all the teeth present.

3. The unique geometry of the curve representing superior dental arch form is approximated by a closed curve with trifocal elliptic properties, with the teeth occupying only a portion of the total curve at its constricted end.

4. The primary determinants of arch form morphology are the (muscle) tissue forces of the resting state in contradistinction to the intermittent forces of muscles in functioning states.

5. The geometry of the curve of dental arch form is so related with the resting forces of the tongue that $PR = C$, where:

P = Pressure/unit area.
R = A radius of curvature at a point along the compound curve corresponding precisely with the pressure site.
C = A mathematic constant, exhibiting variation in magnitude between individuals, and variation in the same individual at different physiologic ages.

6. Considering the circumoral structures as an elastic envelope, the lips and cheeks exert counterbalancing inward tensions against the teeth according to an equation describing forces across the surface of any elastic container, given by:

$$Pi = Pe + T\left(\frac{1}{R} + \frac{1}{R'}\right) \text{ where:}$$

Pi = Internal forces.
Pe = External forces.
T = Tension of the elastic envelope.
R = Radius of curvature in the horizontal plane.
R′ = Radius of curvature in the transverse plane.

7. At least two separate constraints modulate the horizontal form of the dental arch: the mathematic constant of the tongue muscle energy, designated C, (No. 5 above), and the elastic tension of the lips and cheeks, designated T (No. 6 above).

8. Arch form characteristics are such that the form is stabilized and dental equilibrium is attained wherever $C = T$. Transitional or temporary variations in the relationship between C and T (for example, $C<T$, or $C>T$) can alter arch form during growth or pathologic incursions.

It has been the purpose of this thesis to present theoretical relationships between the inherent morphologic architecture of natural dental arch forms and the observed force systems of their environment and, further, to attempt to describe some of the major determinants of superior dental arch form with mathematical statements of physical law.

From the present study, there have evolved innovations in concepts about the geometric characteristics of superior dental arch forms and the presently useful clinical tool of arch form guides for orthodontic therapy.

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